ARTICLE NO. 79-0831R

J80-124

Analytical Model Improvement Using Modal Test Results

40006

Jay C. Chen* and John A. Garba†
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, Calif.

A matrix perturbation method is proposed to calculate the Jacobian matrix and to compute the new eigendata for the parameter estimation procedure. The advantages of the method are the applicability to large complex structures without knowing the analytical expressions for the mass and stiffness matrices, and a cost effective approach for the recomputation of the eigendata. This method also allows the use of other measurements such as modal forces, kinetic energy distribution, and strain energy distributions in the estimation procedure. A realistic sample problem is presented to demonstrate the effectiveness of the proposed method.

Introduction

A PRINCIPAL objective of the modal test is to verify the analytical model by comparing the experimentally determined modal characteristics such as the natural frequencies and mode shapes to those obtained from the analytical model. Frequently, the analytical model does not initially produce characteristics that concur with test results, and consequently the analytical model is adjusted until the analysis and test results agree. Historically, the model adjustment has been accomplished by a "trial and error" approach which was heavily dependent upon the individual's experience and intuition. With increasing complexity of the structural system, the model adjustment becomes difficult and systematic approaches are necessary. In recent years, a number of systematic procedures have been developed[1]; none has received general acceptance, due to certain shortcomings, although many have been successfully applied to specific problems. A review of the previous research and existing procedures can be found in Ref. 2.

In principle the model updating procedure uses the differences between the measured and analytically obtained eigenvectors and eigenvalues to identify or estimate those model parameters that affect these quantities. Therefore, the sensitivity of eigenvalues and eigenvectors with respect to the model parameters is required. Usually it is in the form of a Jacobian matrix whose elements are the partial derivatives of eigenvalues and modal displacements with respect to model parameters. Although methods for evaluating these derivatives are well developed,[3,4] the major difficulty is the establishment of relationships between the parameters under consideration and the individual elements in the mass and stiffness matrices of the model. For simple structures such as spring-mass systems and beams, the analytical formulations of mass and stiffness matrices are readily available; for complex structures, however, the influence of certain parameters on the mass and stiffness elements of the model is not obvious. Furthermore, if the model employs a consistent mass matrix or modal synthesis techniques, relationships between the parameters and the elements of the mass and stiffness matrices are even more complicated. This seems to be the reason why so many reports on the subject matter present only simple structures as sample problems.

Another problem which inhibits the application of the estimation procedure to large complex structures is the reanalysis of eigenvalues and eigenvectors. Basically, correlating and updating the analytical model to match test results is an iterative cycle. After the model is adjusted, a new set of eigenvalues and eigenvectors are computed and compared with the test results. The procedure is repeated until a predetermined convergence between the analysis and test is achieved. For complex structures with large number of degrees-of-freedom, the eigenvalue and eigenvector computation is expensive. In fact, it is the most costly computation within the iteration for most of the procedures under consideration. The objective of the present study is to propose a method by which aforementioned difficulties can be alleviated. The method will be demonstrated on a realistically complex structural system.

Approach for Parameter Estimation

The approach used in the present study is to establish a method which will estimate the parameters of a finite element model capable of providing modal characteristics consistent with those measured in test. The procedure uses the values of parameters originally assigned to the model as a starting point from which the parameters are modified iteratively based on the differences between the analytical and test values. Therefore, the relationship between the measurables and the parameters must be established. The derivation for obtaining this relationship will be briefly described.

Consider a mathematically well-behaved function f of n variables $(r_1, r_2, \ldots, r_n)$, the relationship between the function f and the variable r_n can be written in the form of a Taylor series expansion:

$$f(r_1,r_2,\ldots,r_n) = f(\bar{r}_1,\bar{r}_2,\ldots,\bar{r}_n) + \sum_{i=1}^{n}\left(\frac{\partial f}{\partial r_i}\right)_{r_i=\bar{r}_i}(r_i-\bar{r}_i) + \sum_{i=1}^{n}\left(\frac{\partial^2 f}{\partial r_i^2}\right)_{r_i=\bar{r}_i}(r_i-\bar{r}_i)^2+\ldots \qquad (1)$$

For the small difference between r_i and $\bar{r}_i$, Eq. (1) can be approximated as:

$$f(r_1,r_2,\ldots,r_n) \cong f(\bar{r}_1,\bar{r}_2,\ldots,\bar{r}_n) + \sum_{i=1}^{n}\left(\frac{\partial f}{\partial r_i}\right)_{r_i=\bar{r}_i}(r_i-\bar{r}_i) \qquad (2)$$

If a number of functions are involved, Eq. (2) can be written in matrix form,

$$\{f(r)\} = \{f(\bar{r})\} + \left[\frac{\partial f(r)}{\partial r}\right]_{r=\bar{r}}\{r-\bar{r}\} \qquad (3)$$

Received March 12, 1979; presented as Paper 79-0831 at the AIAA/ASME/ASCE/AHS 20th Structures, Structural Dynamics and Materials Conference, St. Louis, Mo., April 4-6, 1979; revision received Oct. 2, 1979.

Index category: Structural Dynamics.

*Member of Technical Staff. Member AIAA.

†Supervisor, Structures and Dynamics Technology Group. Member AIAA.

where

$$\{f(r)\} = \begin{Bmatrix} f_1(r_1, r_2, \ldots, r_n) \\ f_2(r_1, r_2, \ldots, r_n) \\ \vdots \\ f_m(r_1, r_2, \ldots, r_n) \end{Bmatrix} \tag{4}$$

$$\{f(\bar{r})\} = \begin{Bmatrix} f_1(\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n) \\ f_2(\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n) \\ \vdots \\ f_m(\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n) \end{Bmatrix} \tag{5}$$

$$\left[\frac{\partial f}{\partial r}\right]_{r=\bar{r}} = \begin{bmatrix} \left.\frac{\partial f_1}{\partial r_1}\right|_{r_1=\bar{r}_1} & \left.\frac{\partial f_1}{\partial r_2}\right|_{r_2=\bar{r}_2} \cdots & \left.\frac{\partial f_1}{\partial r_n}\right|_{r_n=\bar{r}_n} \\ \left.\frac{\partial f_2}{\partial r_1}\right|_{r_1=\bar{r}_1} & \left.\frac{\partial f_2}{\partial r_2}\right|_{r_2=\bar{r}_2} \cdots & \left.\frac{\partial f_2}{\partial r_n}\right|_{r_n=\bar{r}_n} \\ \vdots & & \vdots \\ \left.\frac{\partial f_m}{\partial r_1}\right|_{r_1=\bar{r}_1} & \cdots\cdots\cdots\cdots & \left.\frac{\partial f_m}{\partial r_n}\right|_{r_n=\bar{r}_n} \end{bmatrix} \tag{6}$$

$$\{r-\bar{r}\} = \begin{Bmatrix} (r_1-\bar{r}_1) \\ (r_2-\bar{r}_2) \\ \vdots \\ (r_n-\bar{r}_n) \end{Bmatrix} \tag{7}$$

For a structural system under condition, the parameters r_i are to be identified and $\bar{r}_i$ are the corresponding values used in the original analysis. Functions $f_i(r)$ represent the measured eigenvalues and modal displacements which are functions of the parameters r_i, the $f_i(\bar{r})$ are the corresponding eigenvalues and modal displacements obtained from the original model where parameter values of $\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n$ are used. Finally, the matrix $[\partial f/\partial r]_{r=\bar{r}}$ is referred to as the Jacobian matrix whose elements are the derivatives of the eigenvalues and modal displacements with respect to the parameters and their derivatives are evaluated at $\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n$.

Equation (3) can be simplified to read:

$$\{\Delta f\} = \left[\frac{\partial f(r)}{\partial r}\right]\{\Delta r\} \tag{8}$$

where

$$\{\Delta f\} = \{f(r)\} - \{f(\bar{r})\} \text{ and } \{\Delta r\} = \{r-\bar{r}\} \tag{9}$$

Equation (8) states that the differences between the measured quantities and the corresponding analytically obtained quantities are linearly related to the changes of parameters. It must be noted that only in exceptional cases are the eigenvalues and eigenvectors of a structural system linear functions of the parameters that are, in this case, the mass and stiffness properties. However, Eq. (8) as well as Eq. (3) are valid, in an approximate sense, as long as the changes to the parameters are small. Therefore, this is an *a priori* condition for most estimation problems.

In principle the parameter difference $\{\Delta r\}$ can be solved using Eq. (8) once the Jacobian matrix and the vector $\{\Delta f\}$ are available. However, this is true only if the matrix $[\partial f/\partial r]$ is nonsingular and the vectors $\{\Delta f\}$ and $\{\Delta r\}$ have the same dimensions. Unfortunately, the dimensions of the two vectors are usually not equal and other techniques must be used to estimate $\{\Delta r\}$. One well-known technique is the least-squares fit method which is commonly used if the dimension of $\{\Delta r\}$ exceeds that of $\{\Delta f\}$ (Ref. 5). The least-square principle states that $\{\Delta r\}$ should be chosen such that the sum of the squared components of the residual (or error) vector is minimized. Therefore, one may define

$$\{\Delta\} = \{\Delta f\} - \left[\frac{\partial f}{\partial r}\right]\{\Delta r\} \tag{10}$$

as the residual vector; the least squares method seeks to minimize the scalar square error function defined as

$$Q = \{\Delta\}^T\{\Delta\} \tag{11}$$

The minimization requires that the partial derivatives of Eq. (11) with respect to $\{\Delta r\}$ vanish.

$$\frac{\partial Q}{\partial(\{\Delta r\})} = 2\left[\frac{\partial f}{\partial r}\right]^T\left(\left[\frac{\partial f}{\partial r}\right]\{\Delta r\} - \{\Delta f\}\right) = 0 \tag{12}$$

and finally

$$\{\Delta r\} = \left(\left[\frac{\partial f}{\partial r}\right]^T\left[\frac{\partial f}{\partial r}\right]\right)^{-1}\left[\frac{\partial f}{\partial r}\right]^T\{\Delta f\} \tag{13}$$

Based on this basic formulation, variations of Eq. (13) have been developed such as the weighted least square estimation, the minimum variance estimation and the maximum likelihood estimation. In these estimation techniques, the measure of confidence of the parameters and/or the observations is introduced into the estimation procedure by their statistical distributions.[6,7] These methods have been successfully applied to some realistic parameter estimation problems.[8,9]

However, in the present investigation the following facts from the authors' experience will be taken into consideration.

1) Usually the predictions by the original model are fairly close to the experimental measurements, therefore, we are talking about small differences between the original model and the updated model.

2) It is difficult to attach a confidence level or an error distribution in the parameters and the observations. Especially for the test measurements, only those data which are considered 100% accurate are taken into consideration for parameter estimation, since it is felt that contaminated measurements may give totally erroneous results.

Based on these two considerations, the parameter estimation problem in the present investigation will be stated as follows: The number of parameters is greater than the number of observations which are considered to be equally accurate, and the smallest possible parameter changes are sought among the infinite number of possibilities. Then Eq. (8) will be rewritten as:

$$\left.\begin{aligned} A_{11}\Delta r_1 + A_{12}\Delta r_2 + \ldots + A_{1n}\ \Delta r_n &= \Delta f_1 \\ A_{21}\Delta r_1 + A_{22}\Delta r_2 + \ldots + A_{2n}\ \Delta r_n &= \Delta f_2 \\ \vdots \qquad \vdots \qquad\qquad \vdots \quad &\ \ \vdots \\ A_{m1}\Delta r_1 + A_{m2}\Delta r_2 + \ldots + A_{mn}\Delta r_n &= \Delta f_m \end{aligned}\right\} \tag{14}$$

where

A_{ij} = elements of the Jacobian matrix
n = number of parameters
m = number of observations

$$n > m \tag{15}$$

A positive quantity Q will be defined as

$$Q=\Delta r_1^2+\Delta r_2^2+\ldots+\Delta r_n^2 \tag{16}$$

We are seeking a set of Δr_i that satisfy Eq. (14) and also give a minimum Q. From Eq. (14) one may observe that

$$\left\{\begin{matrix}\Delta r_1\\ \Delta r_2\\ \vdots\\ \Delta r_m\end{matrix}\right\}=f(\Delta r_{m+1},\ \Delta r_{m+2},\ldots\Delta r_n) \tag{17}$$

Therefore the quantity Q is also a function of $(\Delta r_m, \Delta r_m + 1,\ldots,\Delta r_n)$. The minimization of Q can be established by letting

$$\frac{\partial Q}{\partial\Delta r_{m+1}}=\frac{\partial Q}{\partial\Delta r_{m+2}}=\ldots=\frac{\partial Q}{\partial\Delta r_n}=0 \tag{18}$$

Equation (14) contains m equations and Eq. (18) contains $(n-m)$ equations, these two sets of equations will determine the n unknowns, i.e., Δr.

After the parameters are estimated based on the abovementioned procedure, new eigenvalues and eigenvectors are calculated. These newly calculated values will then be considered as the original analytical values and the entire procedure is repeated until reaching the convergence.

Matrix Perturbation Technique

One important step in the procedure is the evaluation of the Jacobian matrix whose elements are the derivatives of eigenvalues and eigenvectors with respect to the parameters as expressed by Eq. (6). As mentioned before the difficulty is that the relationship between the parameters and the elements of the mass and stiffness matrices are not explicit for complex structures. For further clarification, Eq. (6) will be rewritten as:

$$\left[\frac{\partial f}{\partial r}\right]=\begin{bmatrix}\left[\frac{\partial\lambda}{\partial M}\right] & \left[\frac{\partial\lambda}{\partial K}\right]\\ \left[\frac{\partial\phi}{\partial M}\right] & \left[\frac{\partial\phi}{\partial K}\right]\end{bmatrix}\begin{bmatrix}\frac{\partial M}{\partial r}\\ \frac{\partial K}{\partial r}\end{bmatrix} \tag{19}$$

where λ and ϕ are eigenvalues and modal displacements, respectively. Their derivatives with respect to elements in the mass and stiffness matrices, as expressed in the first matrix on the righthand side of Eq. (19), can be evaluated with well developed techniques.[10,11]

It is the second matrix whose elements $\partial M/\partial r$, $\partial K/\partial r$ are difficult to obtain for complex structures since the influence of a particular parameter on the elements in the mass and stiffness matrices is not easily derived analytically. Furthermore, if the model employs a consistent mass matrix or modal synthesis techniques for substructuring, the effect of the physical parameters, such as material properties, is distributed in the mass and stiffness matrices that may or may not possess physical characteristics. Except for simple structures such as beams where a lumped mass and an analytical stiffness matrix are available, the Jacobian matrix is, in general, not readily evaluated. Therefore, a matrix perturbation technique will be proposed in this study to evaluate the Jacobian matrix. Also, the same technique can be applied to recomputation of eigenvalues and eigenvectors for the iteration procedure. The perturbation method is a well developed mathematical technique and its applications to structural analysis have been established.[12,13] The technique will be briefly summarized in the following.

If the mass and stiffness matrices of a structural system can be expressed as

$$[M]=[M_0]+\Delta[M] \text{ and } [K]=[K_0]+\Delta[K] \tag{20}$$

respectively, where Δ is a small parameter, then one may postulate that $[M]$ and $[K]$ are analytical functions of the small parameter Δ. Since the eigenvectors $[\phi]$ and eigenvalues $[\lambda]$ are functions of $[M]$ and $[K]$, they can also be expanded into analytical functions of Δ as

$$[\phi]=[\phi_0]+\Delta[\phi_1]+\Delta^2[\phi_2]+\ldots \tag{21}$$

and

$$[\lambda]=[\lambda_0]+\Delta[\lambda_1]+\Delta^2[\lambda_2]+\ldots \tag{22}$$

where $[\phi_0]$ and $[\lambda_0]$ are the analytical eigenvectors and eigenvalues of the $[M_0]$, $[K_0]$ system. Furthermore, the analytical eigenvectors $[\phi_0]$ form a complete orthonormal set in N space, thus any vector in that N space may be represented as a linear combination of the vectors from the $[\phi_0]$ set.

$$\Delta\{\phi_1\}_i=\alpha_{i1}\{\phi_0\}_1+\alpha_{i2}\{\phi_0\}_2+\ldots+\alpha_{in}\{\phi_0\}_n \tag{23}$$

where $\Delta\{\phi_1\}_i$ is the ith vector in the $\Delta[\phi_1]$ set and $\{\phi_0\}_j$ is the jth eigenvector of the $[M_0]$, $[K_0]$ system. Equation (23) can be written in matrix form as

$$\Delta[\phi_1]=[\phi_0][\alpha] \tag{24}$$

Substituting Eqs. (20-23) into the following orthogonality conditions

$$[\phi]^T[M][\phi]=[I] \text{ and } [\phi]^T[K][\phi]=[\lambda] \tag{25}$$

and using the orthogonality conditions for the $[M_0]$, $[K_0]$ system one obtains

$$[\alpha]+[\alpha]^T=-[\phi_0]^T[\Delta M][\phi_0]+O(\Delta^2)$$

$$[\Delta\lambda_1]=[\lambda_0][\alpha]+[\alpha]^T[\lambda_0]+[\phi_0]^T[\Delta K][\phi_0]+O(\Delta^2) \tag{26}$$

For small perturbations in $[\Delta M]$ and $[\Delta K]$, the second order terms $O(\Delta^2)$ in Eqs. (21), (22), and (26) are neglected.

Now for given perturbations in mass and stiffness matrices as shown in Eq. (20), the new updated eigenvalues and eigenvectors can be obtained approximately as expressed in Eq. (26) without having to solve the usual eigenproblem solutions. Therefore, this technique is ideally suited for the recomputation of eigenvalues and eigenvectors after the parameters are estimated and the new mass and/or stiffness matrices are constructed.

As for the eigenvalue and eigenvector derivatives with respect to the specified parameter, a small change in the specified parameter Δr_i will be assumed. This change will produce a new mass or stiffness matrix which is different from the original ones by $[\Delta M]$ or $[\Delta K]$. Using matrix perturbation techniques, the changes in eigendata $\Delta\lambda_i$ and $\Delta\phi_{ij}$ can be calculated. Then in the limiting case, one obtains the derivatives as

$$\frac{\partial\lambda_i}{\partial r_i}=\lim_{\Delta r_i\to 0}\left(\frac{\Delta\lambda_i}{\Delta r_i}\right) \text{ and } \frac{\partial\phi_{ij}}{\partial r_i}=\lim_{\Delta r_i\to 0}\left(\frac{\Delta\phi_{ij}}{\Delta r_i}\right) \tag{27}$$

The advantage of the approach is that explicit or analytical relationships between the parameters and elements in the mass and stiffness matrices are not necessary. Instead, new mass or stiffness matrices must be constructed for the small changes in every parameter and new eigenvalues and eigenvectors will be obtained by the perturbation technique. The derivatives are obtained from Eq. (27). This approach can be easily applied to complex structures.

It should be noted that the matrix perturbation technique is an approximate method since the terms of $O(\Delta^2)$ and higher have been neglected. However, these neglected terms are of the same order of those higher order terms neglected in the Taylor series expansion. In other words, the approximations made in the matrix perturbation technique are consistent with the assumptions made in the parameter estimation procedure. Another important point is that Eq. (24) is true only if the complete $[\phi_0]$ is used. Any truncation of $[\phi_0]$ will make Eq. (24) an approximation. However, experiences indicate if all the fundamental modes are included, sufficient accuracy can be obtained.

Sample Problem

The propulsion subsystem of the Viking Orbiter spacecraft was selected to demonstrate the parameter estimation procedure proposed in the present study. This propulsion subsystem is an integral part of the Viking Orbiter structural system whose mathematical model was of vital importance for the subsequent loads analysis. The results of carefully conducted static and modal tests were used to verify the mathematical model and indeed a very good agreement between the test results and the model predictions were achieved. An attempt was made to further impove the model by correlating the test results with the analytical predictions,[14] however, inconclusive results were obtained.

Figure 1 is a photograph of the propulsion subsystem used in the modal test. It mainly consists of two large propellant tanks, a pressurant tank, a rocket engine, a propellant control assembly and structural support members. The weight of the propellants, fuel and oxidizer, constitutes over 60% of the weight of the entire orbiter spacecraft. Each of the fuel and oxidizer mass was represented by a single mass located at its center of gravity with six degrees-of-freedom. Thus, the total fluid mass of the structural system is represented by a 12×12 matrix. The elements of this fluid mass matrix have been estimated based on the full scale single tank slosh test in which the effective mass was measured under the desired conditions.

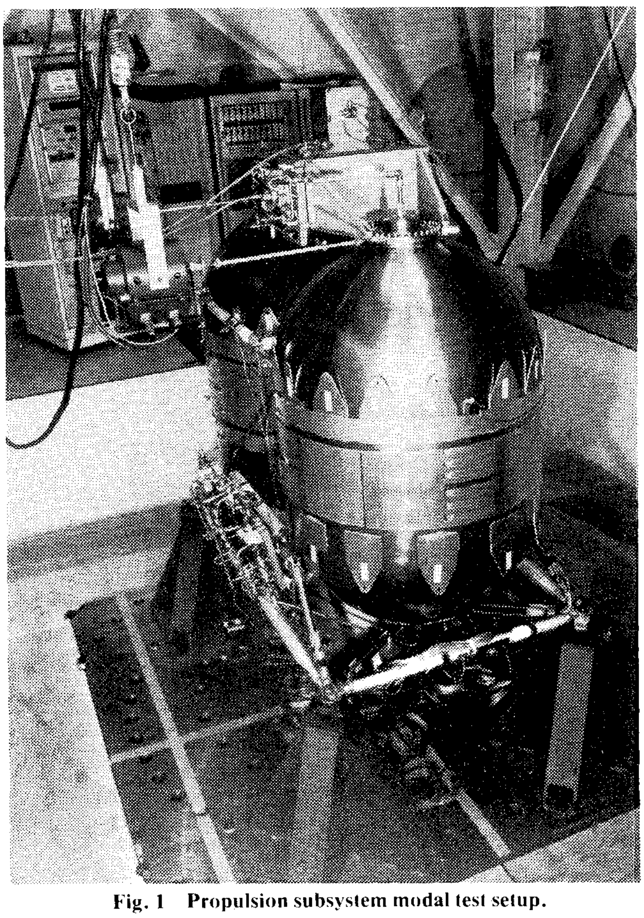

Fig. 1 Propulsion subsystem modal test setup.

Table 1 Description of observations and parameters

Model	Observations (measurements)		Parameters
I	4 frequencies	Oxidizer:	M_x, M_y, M_z
		Fuel:	M_x, M_y
II	5 frequencies	Oxidizer:	M_x, M_y, M_z, I_{xx}
		Fuel:	M_x, M_y, M_z, I_{xx}
III	5 frequencies	Oxidizer:	$M_x, M_y, M_z, I_{xx}, I_{yy}$
		Fuel:	$M_x, M_y, M_z, I_{xx}, I_{yy}$
IV	5 frequencies	Oxidizer:	$M_x, M_y, M_z, I_{xx}, I_{yy}, I_{zz}, I_{xy}$
	2 modes	Fuel:	$M_x, M_y, M_z, I_{xx}, I_{xy}$
		Shear tie:	K
V	5 frequencies	Oxidizer:	$M_x, M_y, M_z, I_{xx}, I_{yy}, I_{zz}, I_{xy}$
	3 modes	Fuel:	$M_x, M_y, M_z, I_{xx}, I_{yy}, I_{xy}$
		Shear tie:	K
VI	5 frequencies	Oxidizer:	$M_x, M_y, M_z, I_{xx}, I_{yy}, I_{zz}, I_{xy}$
	4 modes	Fuel:	$M_x, M_y, M_z, I_{xx}, I_{yy}, I_{zz}, I_{xy}$
		Shear tie:	K

Figure 2 shows the schematic of the analytical model and its coordinate system. The analytical model contains 677 static (stiffness) degrees-of-freedom and 84 dynamic (mass) degrees-of-freedom. There are 117 plate elements, 184 beam elements, and a total of 5 types of propellant tank tab super elements.

From the comparison of the static test results and the corresponding analytical predictions, it was clear that the stiffness representation of the model is fairly accurate. Seemingly, whatever the discrepancies in the modal test results and its analytical prediction must come from the inertia representation of the model. Especially in view of the fact that a large quantity of fluid mass in an elastic tank has been represented by a single large point mass. After all not only the amount of mass but also the distribution of the mass is important in the outcome of the dynamic results. For this reason, the inertia properties of the two masses are chosen as

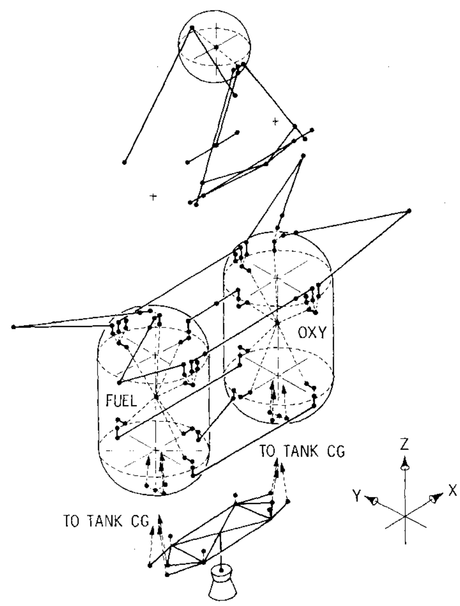

Fig. 2 Schematic of the Viking propulsion subsystem finite element model.

Table 2 Estimated parameter values ($\Delta r/r$)

	Oxidizer							Fuel							Shear Tie
Model	M_x	M_y	M_z	I_{xx}	I_{yy}	I_{zz}	I_{xy}	M_x	M_y	M_z	I_{xx}	I_{yy}	I_{zz}	I_{xy}	K
I	-0.386	-0.340	-0.132					-0.199	0.648						
II	-0.383	-0.334	-0.145	-0.048				-0.179	0.639	0.027	0.055				
III	-0.355	-0.334	-0.153	-0.048	-0.100			-0.161	0.639	0.021	0.056	0.044			
IV	0.045	-0.404	-0.292	-0.131	-0.675	-0.172	-0.513	-0.897	0.521	0.043	0.011	0.170			-0.510
V	0.067	-0.473	-0.286	-0.049	-1.769	0.512	-0.694	-0.174	0.677	-0.039	0.057	-0.931		0.234	-0.160
VI	0.057	-0.470	-0.283	-0.054	-1.793	0.476	-0.676	-0.133	0.663	-0.041	0.054	-0.948	0.018	0.195	-0.183

Table 3 Frequency comparison (Hz)

			Updated Model					
Description	Test	Original Model	I	II	III	IV	V	VI
Oxidize & Fuel in Y, in Phase	12.95	12.37	12.95	12.95	12.95	12.97	12.86	12.88
Oxidizer in Z	17.66	15.85	17.66	17.66	17.66	17.69	17.58	17.58
Oxidizer & Fuel in Z	20.80	19.37	20.80	20.80	20.80	20.81	20.78	20.78
Oxidizer & Fuel in Y, Out-of-Phase	22.97	26.49	22.97	22.97	22.97	23.11	22.52	22.57
Fuel in Z	28.33	27.90	28.55	28.33	28.33	28.33	28.33	28.33
Pressurant Tank in θ_y	32.76	34.19	33.74	33.74	33.74	33.81	33.74	35.58
Pressurant Control Assembly in X	42.80	42.06	42.02	42.02	42.02	42.06	42.03	42.03
Pressurant Tank in Y	50.67	47.69	47.66	47.67	47.66	47.84	47.66	47.65
Pressurant Control Assembly in Z	50.40	52.39	52.43	52.43	52.43	52.41	52.40	52.40
Pressurant Control Assembly in θ_y	65.38	60.96	61.22	61.22	61.20	61.57	61.18	61.16
RSS Error		7.29	5.67	4.80	5.30	5.31	5.70	6.30

parameters to be estimated by the modal test results. Six different models with different combinations of parameters are estimated from different combinations of observations. However, the number of parameters is always greater than the number of observations used such that the minimization procedure outlined previously can be applied. Table 1 defines these six models. The first three models use only the frequency measurements as the observations. The next three models use the frequency measurements as well as the mode shape measurements. Also, the stiffness of the shear tie connecting the lower portion of the two tanks was included since the previous study[12] had found that the shear tie played an important role in one of the modes.

Since only the propellant inertia properties are to be estimated, the observations, frequencies, and mode shapes are limited to those predominately tank modes. Table 2 shows the results of the estimated parameters for these six models. They are expressed as the ratio of the incremental values with respect to the original values. In models V and VI, the incremental inertia I_{yy} exceeds the original value negatively. Seemingly, this is erroneous since the moment of inertia is negative. However, the situation is such that for a single mass representation of a large quantity of moving fluid, a negative inertia may be reasonable. It is also clear that models I, II and III have similar estimates for the parameters and models IV, V and VI possess different estimates for the parameters. The differences are obviously due to the consideration of mode shapes for the observations in the later models. Within these two groups, the parameters in different models seem to converge to common values.

Using these estimated parameters, the perturbed mass and/or stiffness matrices are constructed. Matrix perturbation techniques are applied to obtain the new eigenvalues and eigenvectors for the six updated models. Table 3 shows the frequency comparisons of these models with respect to the test measured frequencies and the original model predicted frequencies. Noticeable improvements are obtained for these modes dominated by the propellant tank motion. Also, those models obtained by using mode shapes as observations provide lesser improvements in frequency than those using frequency only. This is quite reasonable. Next, the mode shapes of the updated models will be compared.

Table 4 shows the comparison of the measured model displacements of certain degrees-of-freedom and the corresponding values from the original and the updated models. For this particular mode, the updated models

Table 4 Mode shape comparison for mode 2

Description of DOF	Test	Original Model	Updated Model					
			I	II	III	VI	V	VI
Engine in X	0.5008	0.5639	0.5138	0.5182	0.5219	0.4908	0.4976	0.4969
Oxidizer in X	0.2093	0.2470	0.2100	0.2131	0.2164	0.2270	0.2340	0.2331
Oxidizer in Z	0.2724	0.2281	0.2584	0.2535	0.2489	0.3051	0.2916	0.2928
Fuel in X	0.2039	0.2248	0.2007	0.2028	0.2049	0.1906	0.2014	0.2015
Pressurant Tank in X	-0.1614	-0.1039	-0.1425	0.1391	0.1336	0.0851	0.1182	0.1178
RSS Error		0.1054	0.0271	0.0343	0.0426	0.0865	0.0535	0.0539

Table 5 Modal force comparison for mode 1

Member No.	Test	Original Model	Updated Model					
			I	II	III	IV	V	VI
3	-66.8	-73.8	-81.3	-81.1	-81.0	-78.1	-79.2	-79.2
4	-568.2	-572.9	-549.1	-549.3	-549.3	-552.7	-549.3	-546.4
8	-292.1	-302.4	-296.7	-296.7	-296.6	-296.0	-293.4	-293.9
11	151.4	152.6	159.7	159.8	160.0	159.3	160.2	160.4
12	497.7	549.1	522.7	522.6	522.6	527.3	520.2	521.2
18	264.9	309.1	305.2	305.2	305.3	306.6	305.0	305.4
36	-452.6	-489.0	-493.8	-493.3	-493.0	-487.2	-485.4	-486.0
37	-197.7	-130.7	-117.6	-118.1	-118.3	-123.4	-120.9	-121.2
40	205.9	249.2	233.6	233.4	233.1	232.9	228.0	228.6
41	476.6	427.2	435.6	435.9	436.3	437.6	438.5	439.0
RSS Error		122.1	116.0	115.3	114.9	109.7	107.8	108.6

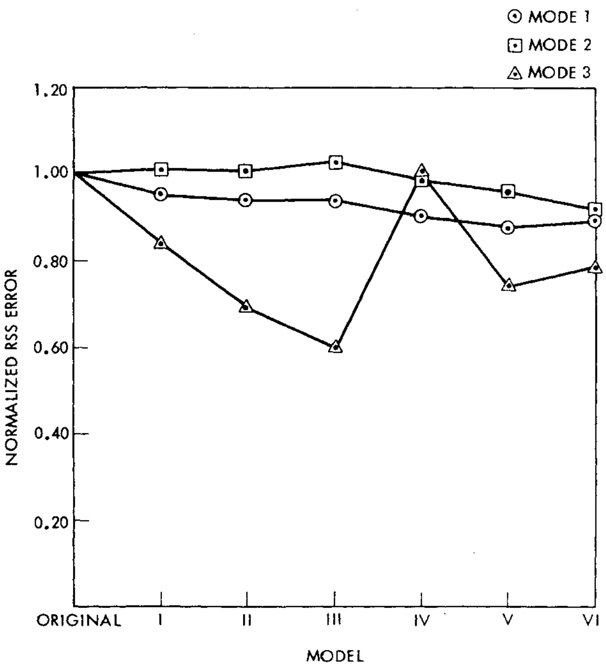

Fig. 3 rss error for updated mode shape.

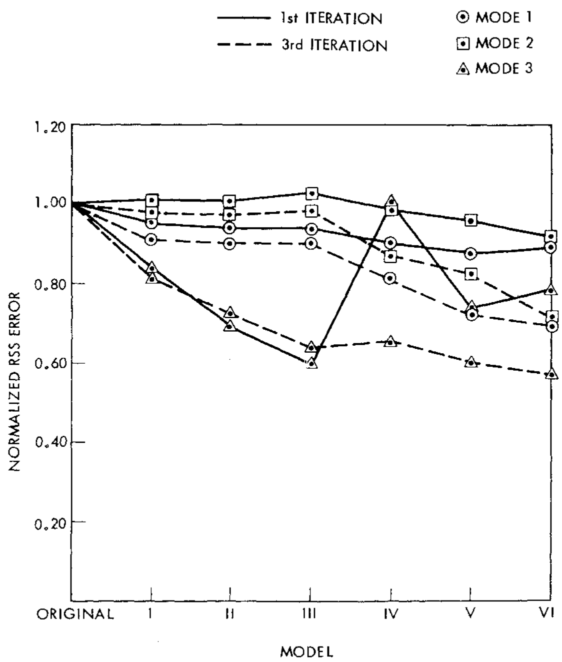

Fig. 4 rss error for updated modal force.

provided better comparisons than the original model. Figure 3 shows the trend of the improvements of mode shapes from the updated models by plotting the rss (root-sum-square) error normalized by the rss error of the original model for the six models. Using this criteria the mode shapes for mode 1 and mode 3 of the updated models are worse than that of the original model. However, the modal displacements of very few degrees-of-freedom, whose amplitudes are relatively large, have been considered in this plot. Therefore Fig. 3 may not be reflecting the true improvements of the updated models. A better way to evaluate the improvement or the lack of it is to compare the modal forces. Table 5 shows the comparison of measured modal forces of ten primary structural members and their corresponding values from the original and updated models. As more parameters and observations are considered better correlations, i.e., smaller rss errors, are achieved. Figure 4 shows the trend of the improvements of the modal forces from the updated models. Since the stiffness of the model remains unchanged except for the shear tie, the improvements are from the mode shape changes. Therefore Fig. 4 is a better measurement of improvements for the mode shapes. In Fig. 4 improvements from the subsequent iterations are also shown.

In general, the updated models provide excellent frequency correlations for those modes dominated by the propellant tanks and keep other frequencies virtually unchanged. From the modal force comparison, it is also evident that the mode shapes have been improved, and the improvements are better for the models with more parameters and using more observations.

Conclusions

A matrix perturbation method has been proposed to calculate the Jacobian matrix and to compute the new eigenvalues and eigenvectors for the parameter estimation procedure. The advantages of the method are the applicability to large complex structures without the analytical expressions of mass and stiffness matrices, and the cost effective approach to the re-computation of the eigendata. Also this method allows the use of other measurements such as the modal forces as the observations in the estimation procedure. This increases the flexibility of the procedure.

Although in the same problem the estimated parameters converge to approximate common values in two groups, the selection of parameters and observations still affects the final outcome of the updated models. Perhaps, the subject of parameter and observation selection should be further studied such that a systematic approach can be found.

Acknowledgment

This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract NAS 7-100 sponsored by NASA. The effort was supported by A. Amos and R. Goetz, Materials and Structures Division, Office of Aeronautics and Space Technology, NASA.

References

[1] Pilkey, W. D. and Cohen, R. (eds.), *System Identification of Vibrating Structures, Mathematical Models from Test Data*, American Society of Mechanical Engineers, New York, 1972.

[2] Collins, J. D., Hart, G. C., Hasselman, T. K., and Kennedy, B., "Model Optimization Using Statistical Estimation," J. H. Wiggins Company, Redondo Beach, Calif., Tech. Rept. 1087-2, May 1973.

[3] Taylor, D. L. and Kane, T. R., "Multiparameter Quadratic Eigenvalue Problems," *Journal of Applied Mechanics*, Vol. 42, June 1975, pp. 478-483.

[4] Nelson, R. B., "Simplified Calculation of Eigenvector Derivatives," *AIAA Journal*, Vol. 14, Sept. 1976, pp. 1201-1205.

[5] Sorenson, H. W., "Least-Square Estimation: From Gauss to Kalman," *IEEE Spectrum*, July 1970, pp. 63-68.

[6] Lewis, T. O. and Odell, P. L., *Estimation in Linear Models*, Prentice-Hall, Englewood Cliffs, N.J., 1971.

[7] Greville, T.N.E., "The Pseudoinverse of a Rectangular Singular Matrix and Its Application to the Solution of System Linear Equations," *SIAM Review*, Vol. 1, Jan. 1959, pp. 38-43.

[8] Berman, A. and Flannelly, W. G., "Theory of Incomplete Models of Dynamic Structures," *AIAA Journal*, Vol. 9, Aug. 1971, pp. 1481-1487.

[9] Collins, J. D., Hart, G. C., Hasselman, T. K., and Kennedy, B., "Statistical Identification of Structures," *AIAA Journal*, Vol. 12, Feb. 1974, pp. 185-190.

[10] Hart, G. C. and Collins, J. D., "The Treatment of Randomness in Finite Element Modeling," *Transactions of the SAE*, Vol. 81, Pt. 4, 1970, pp. 2408-2416.

[11] Collins, J. D. and Thompson, W. T., "The Eigenvalue Problem for Structural Systems with Statistical Properties," *AIAA Journal*, Vol. 7, April 1969, pp. 642-648.

[12] Chen, J. C. and Wada, B. K., "Matrix Perturbation for Structural Dynamic Analysis," *AIAA Journal*, Vol. 15, Aug. 1977, pp. 1095-1100.

[13] Stetson, K. A. and Palma, G. E., "Inversion of First-Order Perturbation Theory and Its Application to Structural Design," *AIAA Journal*, Vol. 14, April 1976, pp. 454-460.

[14] Garba, J. A. and Wada, B. K., "Application of Perturbation Methods to Improve Analytical Model Correlation with Test Data," SAE Paper 770959, Los Angeles, Calif., Nov. 1977.